Relativity in rotating frames

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Abstract

A common error, originally due to Ehrenfest, is corrected.

A recent book [1] with the same title as above contains 17 articles by various authors, following the English translation of a brief note by Paul Ehrenfest [2] who applied a Lorentz contraction to the rim of a rotating disk, whose radius was not contracted. This is plainly wrong: Lorentz contractions occur when we compare the lengths of segments which are at rest in different inertial frames in relative motion with respect to each other. However, no two points on the rotating disk have the same velocity [3] and therefore they cannot define any Lorentz frame, not even an "instantaneous" one.

In the real world, if we want to impart a constant velocity to a real rod, we can kick it at one end so that the rod acquires a linear momentum. However there are no rigid bodies, there are only elastic or plastic ones. The kick will slowly propagate throughout our rod as elastic vibrations which will gradually damp out, so that the rod finally imitates a Lorentz frame. Likewise, if we kick a disk to give it angular momentum, it will gradually approach the limit of uniform rotation, but then no two points on the disk will have the same velocity, and the rule for Lorentz contraction is not applicable.

What actually happens is easily seen by using rotating coordinates, $\phi' = \phi + \omega t$, in which the disk is "at rest." This is a passive transformation, a mere relabelling of coordinates, not an active transformation, and there is no conceptual difficulty. In that noninertial frame, a centrifugal potential appears, which causes an elatic distortion of the disk. This is a standard problem of elastic equilibrium (or of strength of materials). The theory of relativity is not involved at all.

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REFERENCES

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[3] A. Peres, "Synchronization of clocks in a rotating frame," Phys. Rev. D **18**, 2173–2174 (1978).